Modify Newton Method to Solve Nonlinear Equations by Using Decay Method

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Modify Newton Method to Solve Nonlinear Equations by Using Decay Method

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Abstract. In this paper we will modify Newton method to solve nonlinear equations by another iteration method, say \( x = g(x) \), and using decay method. We will present some conditions of \( g(x) \) which our method will be performance.

Keywords: Newton method; Iteration method; Nonlinear equation; Decay method.

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INTRODUCTION

One of the most important occurring problems in scientific work is to locate a real root of a nonlinear equation

\[ f(x) = 0 \] (1)

Analytic methods for solving such equations rarely exist, and therefore, one can hope to obtain only approximate solution by relying on iteration methods. For a survey of the most important algorithms, some excellent textbooks are available (see [1, 2, 4]). Being quadratic ally convergent, Newton's method (NM) is probably the best-known and most widely used algorithm. Time to time the method has been derived and modified in a variety of ways, one such method derived from Newton's method by approximating the derivative with non-derivative term of difference quotient is Steffensen's method (SM) [3, 5].

MODIFY NEWTON METHOD

For iteration method \( x_{n+1} = g(x_n) \), we know if

\[ g'(\alpha) = g''(\alpha) = \cdots = g^{(k)}(\alpha) = 0, \quad g^{(k+1)}(\alpha) \neq 0 \]

where \( \alpha \) is the solution of (1), then the order of iteration method is \( k + 1 \). So we will write:

\[ g_1(x) = x + \lambda f + \mu xf \]
\[ g_2(x) = x + \lambda f + \mu f^2 \]
\[ g_3(x) = x + \lambda f + \mu f f' \]
\[ g_4(x) = x + \lambda f + \mu xf^2 \]

With \( g_i'(\alpha) = g_i''(\alpha) = 0 \) for \( i = 1, 2, 3, 4 \) and solve equations to finding \( \lambda, \mu \). We will have:
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

(Newton method) for \( i = 1, 3 \) and we will have:

\[ x_{n+1} = x_n - \frac{f}{f'} + \frac{f^2 f''}{2 f'^3} \quad (2) \]

for \( i = 2, 4 \). But if we used the following decay method

\[ G(x) = (1 - \mu - \lambda)x + \mu g(x) + \lambda \left( x - \frac{f(x)}{f'(x)} \right) \]

Where \( x = g(x) \) is an iteration method to solve (1). By \( G'(\alpha) = G''(\alpha) = 0 \) we will get

\[ \mu = \frac{-f''}{(f' g' - f'')'} \quad \text{and} \quad \lambda = \frac{g' f'}{(f' g' - f'')'} \]

It is important that the denominator of two fractions is:

\[ (f'(g' - 1))' \]

That we will lead to chose iteration method and function \( g(x) \), it must be simply to \( g' - 1 \) and also to \( (f'(g' - 1))' \). If we chose

\[ g(x) = x \pm f(x) \]

then we will get Newton method again, and the choices \( g(x) = x \pm f^2(x) \)

send to our:

\[ G(x) = x - \frac{f}{f'} + \frac{f^2 f''}{2 f'^3} \quad (3) \]

And with

\[ g(x) = x \pm xf(x) \]

we will get the method

\[ G(x) = x - \frac{f}{f'} + \frac{f^2 f''}{2 f'^3 + 4 f f' f''} \quad (4) \]

**NUMERICAL EXAMPLE**

In this section, we will present numerical example using MAPLE software in the following table. In this table

\[ f_1(x) = x^6 - 36x^5 + 450x^4 - 2400x^3 + 5400x^2 - 43200x + 725 \]

and * means OVERFLOW or method is nonstop to fix point. But / means method is oscillated and in three-th column output is closer to solution of another
methods. For function \( \sin x \) the method NM go to, \(-4\pi\) and method of equation (3) go to \(-3\pi\) and method of equation (4) go to zero. The numbers in the table are the number of iterations.

### TABLE 1.

<table>
<thead>
<tr>
<th>Function</th>
<th>Start point</th>
<th>NM</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^3 - e^{-x} )</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( \sin x )</td>
<td>1.5</td>
<td>3</td>
<td>*</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>( x^3 + 4x^2 - 10 )</td>
<td>1.5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>( \sin x - 0.5x )</td>
<td>1.5</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( \tan^{-1} x )</td>
<td>2</td>
<td>*</td>
<td>*</td>
<td>8</td>
<td>*</td>
</tr>
<tr>
<td>( 10xe^{-x^2} - 1 )</td>
<td>1</td>
<td>4</td>
<td>*</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>( f_1(x) )</td>
<td>15</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>( x \log^{x}_{10} - 1.2 )</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>( x^2 - 5 )</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

### CONCLUSION

In this paper, we used the decay method to modify Newton method and we get some modifies formula of Newton method. The equation (3) is better than equations but it used secondary diffraction of \( f \).

### ACKNOWLEDGMENTS

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### REFERENCES